Int. J. Heat Mass Trasnfer. Vol. 8, pp. 455–466. Pergamon Press 1965. Printed in Great Britain

EXPERIMENTAL INVESTIGATION OF THERMAL DIFFUSION EFFECTS IN LAMINAR AND TURBULENT SHEAR FLOW

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(*Received* 1 *July* 1964 *and in revised form* 31 *August* 1964)

Abstract—An experimental investigation was conducted to determine the order of magnitude of thermal diffusion across a laminar and a turbulent shear layer. A short length of a cooled free jet was passed through stationary gas and subsequently recaptured into a continuous circulating system. Various mixtures of helium and nitrogen were investigated. With temperatures of 78°K in the jet and 310°K in the surrounding chamber, steady-state helium concentrations in the laminar jet were as much as 7 per cent smaller than in the surroundings. The experimental results are in good agreement with a simplified analysis.

With a turbulent sheat layer between the jet and surroundings, the helium concentration inside the jet increases to within 0·1 per cent higher than the chamber level. The thermal diffusion ratio (i.e. thermal to mass concentration diffusion coefficients) in the turbulent shear layer was thus at least two orders of magnitude smaller than in the laminar case and of opposite sign. It is suggested that similar separation effects are to be expected for other steady flows with closed streamlines, such as base flows and flows past cavities.

NOMENCLATURE

c, mass concentration of helium;
c_m, molar concentration of helium;
$\bar{c}_p$, $c\, c_{p_{He}} + (1 - c)c_{p_{N_2}}$, specific heat of mixture;
c_{p_i}, specific heat of i (i = helium or nitrogen);
D, coefficient of mass diffusion;
f, non-dimensional stream function;
j, mass flow rate due to diffusion;
k, thermal conductivity;
k_T, thermal diffusion ratio;
L, length of jet;
Le, $\rho\bar{c}_p D/k$, Lewis number;
$\dot{m}$, mass flow rate of helium;
M_{21}, molecular weight ratio, $M_{N_2}/M_{He} = 7{\cdot}0$;
n, frequency in concentration meter;
p, pressure;
Pr, $\mu\bar{c}_p/k$, Prandtl number;
r radius;
Re, $\rho_1 u_1 L/\mu_1$, Reynolds number;
T, temperature;
u, v, velocity component in axial and radial directions respectively;
V, output voltage;
x, y, coordinate in axial and radial directions respectively;
δ_c, thickness of concentration layer;
δ_T, thickness of temperature layer;
$\xi, \eta, \bar{\eta}$, similarity variables;
λ, $\rho\mu/\rho_1\mu_1$;
μ, viscosity;
ν, μ/ρ, kinematic viscosity;
ρ, density;
σ, $c(1 - c)/c_m(1 - c_m) = [1 + (M_{21} - 1)c]^2/M_{21}$;
ψ, stream function;
$(\ )_0$, on dividing stream tube;
$(\ )_1$, inside jet;
$(\ )_2$, in the surrounding chamber;
$(\ )^t$, turbulent flow;
$\overline{(\ )}$, mean value, $[(\ )_1 + (\ )_2]/2$.

1. INTRODUCTION

IF A TEMPERATURE gradient is set up in a motionless, homogeneous mixture of gases, the mixture will begin to separate. Usually the lighter component will tend to migrate towards the higher temperature, the phenomenon being known as thermal diffusion. Until a few years ago, it was assumed to be negligible in problems connected with fluid mechanics. However, the large temperature gradients which are encountered in hypersonic boundary layers, together with the suggested use of low density gases to protect the surfaces can result in conditions under which thermal diffusion is no longer negligible. For the case of a laminar boundary layer, theoretical prediction and experiment agree reasonably well [1].

For the turbulent boundary layer, theories such as by Tewfik [2] and Culick [3] usually take into account thermal diffusion only within the laminar sublayer. Tewfik shows that in some cases the effect of thermal diffusion can be fairly high. Furthermore, theoretical predictions do not always agree well with experiments as shown in Culick's Fig. 10 [3]. It was therefore planned to directly determine the order of magnitude of thermal diffusion in turbulent flow by separating an initially homogeneous mixture of gases.

In the laminar case, the magnitude of the thermal diffusion ratio can be computed from the kinetic theory of gases if the force laws between the molecules and the characteristics of the latter are known. It is important to notice that thermal diffusion, being a second order effect, cannot be predicted with "simple" kinetic theories and that there is a strong dependence on the force laws of the interactions between molecules. This suggests that a theoretical prediction of thermal diffusion in turbulent flow would require a detailed understanding of turbulence beyond that now available. Therefore an experiment was conducted to ascertain the order of magnitude of thermal diffusion in *one special kind of turbulence* situation, namely that of a turbulent shear layer. The question of how well such results can be transferred to other kinds of turbulence, e.g. the outer part of a turbulent boundary layer, cannot be answered without further experiment or detailed knowledge of the behavior of turbulence.

The experiment to be described was conducted with a cooled axisymmetric jet. First, a description of the basic experiment as well as an approximate theoretical result for the laminar case will be given. Thereafter the experimental setup and results are presented.

2. DESCRIPTION OF THE BASIC EXPERIMENT

Usually, thermal diffusion effects are small when compared with ordinary mass diffusion. It follows that it is quite difficult to distinguish between the two contributions experimentally, especially if very small effects are anticipated as in the turbulent case. Furthermore the effects in a turbulent boundary layer may be masked by thermal diffusion in the laminar sublayer adjacent to any surface. To circumvent these difficulties the experiment shown in Figs. 1 and 2 was planned. It consists essentially of a closed loop formed by a jet, a blower, a cooler and a nozzle. The gas mixture in this loop is cooled with liquid nitrogen to about 78°K. An outer container surrounds an open section of the loop, it being filled with the same gas mixture and at about room temperature. In steady-state operation, the mass exhausting from the nozzle will be equal to the mass entering the intake, independent of the latter diameter. The part of the cooled loop extending from the nozzle to the intake will therefore be contained within some dividing streamtube as indicated in Fig. 2.

Initially, the entire system, contained a homogeneous mixture of nitrogen and helium. When the mixture inside the jet is relatively cold the temperature gradient imposed over the shear layer gives rise to a thermal diffusion transport of helium across the dividing-streamtube and thus into the outer container. A concentration difference is then built up between the jet and the container, the concentration inside the jet being the smaller. This concentration difference gives rise to a helium transport due to "concentration" diffusion which is directed back into the jet. In the steady state thermal and "concentration" diffusion are in balance and the measured concentration difference between the jet and the container furnishes a direct measure of the relative importance of the two effects.

The mass transport of helium across the dividing streamline is

$$\dot{m} = 2\pi \int_0^L r_0 j_0 \, dx$$

where r_0 is the radius of the dividing streamtube. From kinetic theory the diffusional mass transport across the streamtube surface is

$$j_0 = -\rho_0 D_0 \left[\frac{\partial c}{\partial r} + \sigma \frac{k_T}{T} \frac{\partial T}{\partial r}\right]_0$$

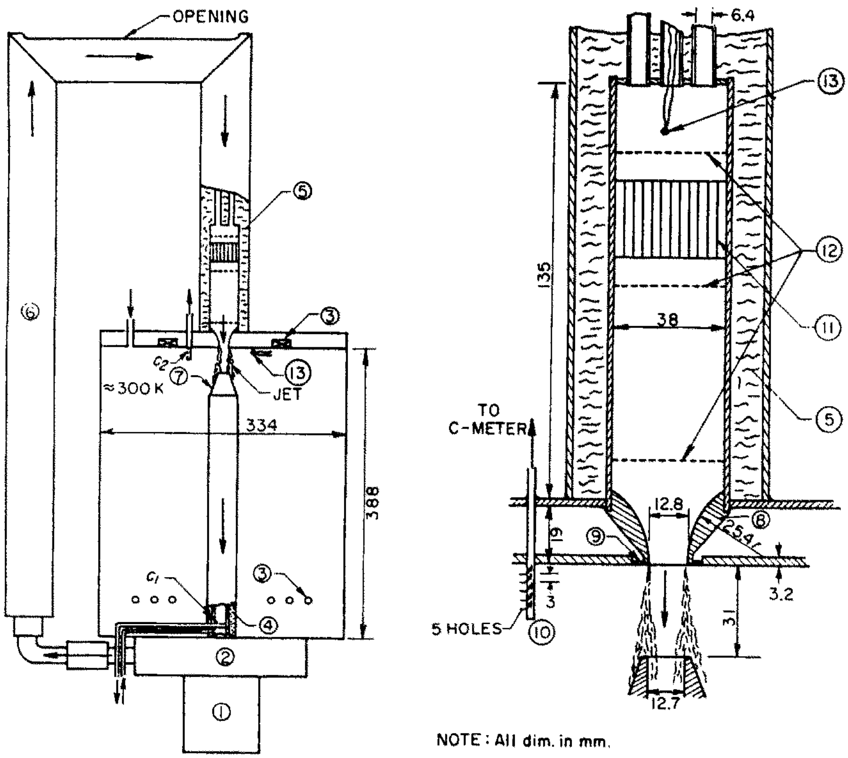

FIG. 1. Experimental arrangement: (a) overall circuit, (b) detail of nozzle and intake: 1. Motor; 2. Blower; 3. Heater; 4. Insulation; 5. Liquid nitrogen; 6. Cooler, 6 tubes, 6·4 mm, i.d.; 7. Intake; 8. Cooled copper nozzle; 9. Lucite ring; 10. Concentration probe, 5 holes; 11. Flow straightener, 3·2 mm D. straws; 12. Screens, 12 mesh/cm, 0·3 mm D. wire; 13. Thermocouple.

Here, ρ is the density, T the temperature, D the mass diffusion coefficient, k_T the thermal diffusion ratio and c the helium concentration. The subscript $(\ )_0$ indicates that all values are taken on the dividing streamtube surface. If it is postulated that the mass transport due to diffusion in turbulent flow is partly due to a concentration gradient and partly due to temperature gradient as in the laminar case, there can be defined analogously,

$$j_0^t = -\rho_0 D_0^t \left[\frac{\partial c}{\partial r} + \sigma \frac{k_T^t}{T}\frac{\partial T}{\partial r}\right]_0$$

Here D^t and k_T^t determine the magnitudes of mass and thermal diffusion, which in detail are determined by the flow field. They must therefore be transferred with some care to other configurations.

For shear layers which are thin compared with r_0, similarity is expected in both the laminar and turbulent cases. As the flux vanishes in the steady state, i.e. $j_0(x) = 0$, there follows

$$\left(\frac{\partial c}{\partial r}\right)_0 = -\sigma_0 \frac{k_{T_0}}{T_0}\left(\frac{\partial T}{\partial r}\right)_0$$

for the laminar case and

$$\left(\frac{\partial c}{\partial r}\right)_0 = -\sigma_0 \frac{k_{T_0}^t}{T_0}\left(\frac{\partial T}{\partial r}\right)_0.$$

for the turbulent case.

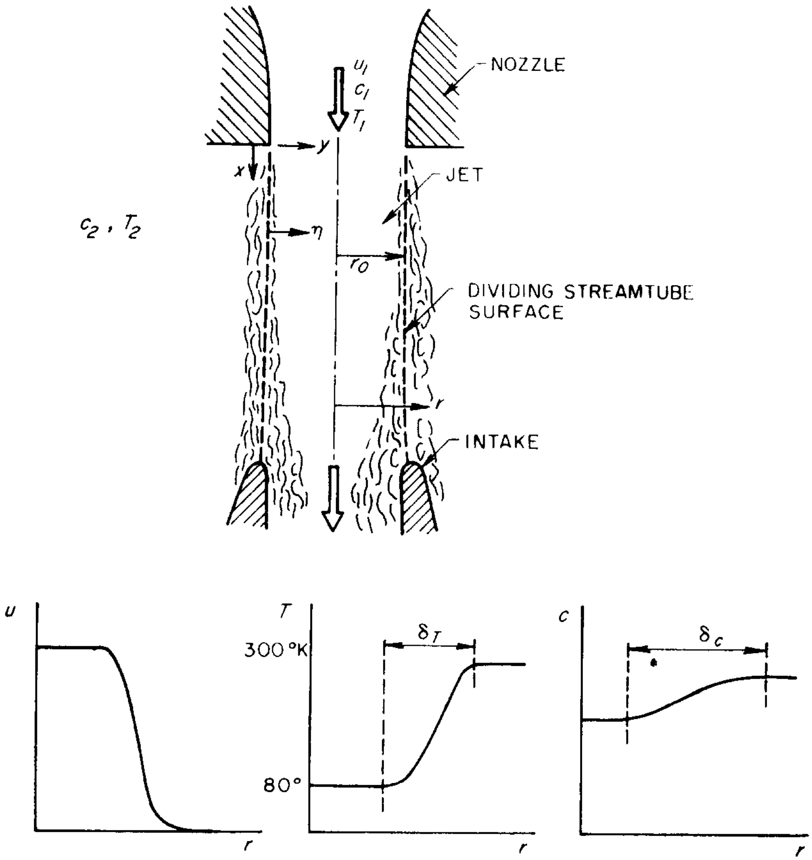

FIG. 2. Detail of jet flow.

An order of magnitude of the separation effect may be found by observing that (see Fig. 2) the gradients may be approximated by the overall changes across the layer thickness, i.e.

$$\left(\frac{\partial c}{\partial r}\right)_0 \cong \frac{c_2 - c_1}{\delta_c}, \left(\frac{\partial T}{\partial r}\right)_0 \cong \frac{T_2 - T_1}{\delta_T}$$

and that the thickness, in turn, vary with the transport coefficients as

$$\delta_c \sim \sqrt{D}, \delta_T \sim \frac{(\sqrt{k})}{\rho \bar{c}_p}$$

$$Le = \frac{\rho \bar{c}_p D}{k}, T_0 \cong \frac{T_1 + T_2}{2}.$$

Here, subscripts 1, 2 are used for the condition in the cold jet and the warm surroundings respectively. The following expressions then result when the flux vanishes

$$c_2 - c_1 \sim -\sigma_0 k_{T_0} \frac{T_2 - T_1}{T_0} (\sqrt{Le}) \text{ (laminar)} \quad (1)$$

$$c_2 - c_1 \sim -\sigma_0 k^t_{T_0} \frac{T_2 - T_1}{T_0} (\sqrt{Le_t}) \text{ (turbulent)} \quad (2)$$

They define a thermal diffusion ratio which can be determined if the remaining quantities are either measured or calculated.

For the turbulent case, the following considerations will give some indication of the effects to be expected. The equation describing the conservation of helium is

$$\frac{\partial(\rho c)}{\partial t} + \frac{\partial(\rho u c)}{\partial x} + \frac{\partial(\rho v c)}{\partial y} = -\frac{\partial j}{\partial x} - \frac{\partial j}{\partial v}.$$

In turbulent flow, the density, velocity components and concentration are time dependent. Using the continuity equation and defining in the usual way

$$\rho(t) = \bar{\rho} + \rho'(t),\ u(t) = \bar{u} + u'(t),$$
$$v(t) = \bar{v} + v'(t)\ c(t) = \bar{c} + c'(t)$$

one obtains for the steady state

$$(\bar{\rho}\,\bar{u} + \overline{\rho' u'})\frac{\partial \bar{c}}{\partial x} + (\bar{\rho}\,\bar{v} + \overline{\rho' v'})\frac{\partial \bar{c}}{\partial y}$$
$$= -\frac{\partial \bar{j}}{\partial x} - \frac{\partial \bar{j}}{\partial y} - \frac{\partial}{\partial x}[\bar{\rho}\,\overline{u'c'} + \bar{u}\,\overline{\rho' c'}]$$
$$-\frac{\partial}{\partial y}[\bar{\rho}\,\overline{v'c'} + \bar{v}\,\overline{\rho' c'}].$$

The fluctuations in j are negligible as j itself is small compared with $\bar{\rho}\,\overline{v'c'}$.

In the case of an homogeneous mixture both the left-hand side and the terms on the right-hand side explicitly containing $c'(=0)$ vanish. Thus turbulent flow in of itself does not give rise to an additional transport in the classical sense, and thus does not separate an initially homogeneous mixture of gases. However, these considerations do not exclude the possibility of an additional mass transport due to a temperature gradient if concentration gradients are present. Of course, it does seem very unlikely that the effects are large compared with ordinary turbulent mixing.

3. ANALYSIS FOR A LAMINAR SHEAR LAYER

As the laminar jet supplied a check on the experimental setup, more detailed analysis was carried out for that case. The equation governing the flow in a shear layer thin compared with the radius of the jet are identical with the boundary layer equations given in reference [4].

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{3}$$

$$\rho u\frac{\partial c}{\partial x} + \rho v\frac{\partial c}{\partial y} = \frac{\partial}{\partial v}\left[\rho D\left(\frac{\partial c}{\partial y} + \frac{k_T}{T}\frac{\partial T}{\partial y}\right)\right] \tag{4}$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{5}$$

$$\rho u\bar{c}_p\frac{\partial T}{\partial x} + \rho v\bar{c}_p\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + u\frac{\partial p}{\partial x} + \rho D\left(\frac{\partial c}{\partial y} + \frac{k_T}{T}\frac{\partial T}{\partial y}\right)(c_{p\mathrm{He}} - c_{p\mathrm{N_2}})\frac{\partial T}{\partial y} \tag{6}$$

representing mass, component mass, momentum and energy conservation respectively.

The length x is measured along the dividing stream tube surface and y normal to it as shown in Fig. 2, u and v being the corresponding velocity components in the x and y direction. If the diameter of the intake is correctly matched to the jet diameter and if the radius of curvature of the intake wall is small compared with the jet diameter, most of the shear layer develops in a region of constant pressure. For the length to diameter ratio of 2·45 used in the present tests, the same is reasonable as well for a slightly mismatched intake diameter. The pressure gradient in the momentum and energy equations then vanish. Introducing the usual similarity transformations

$$\xi = x$$
$$\eta = \left(\sqrt{\frac{u_1}{\nu_1 x}}\right)\int_0^y \frac{\rho}{\rho_1}\,dy$$

and the non-dimensional stream function

$$f = \frac{\psi}{(\sqrt{\nu_1 u_1 \xi})}$$

with

$$u = \frac{\rho_1}{\rho}\frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\rho_1}{\rho}\frac{\partial\psi}{\partial x}$$

then reduces the conservation relations to the following set of ordinary differential equations in which $\lambda = \rho\mu/\rho_1\mu_1$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\lambda\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2}\right) + \frac{f}{2}\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} = 0 \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\left[\lambda\frac{Le}{Pr}\left(\frac{\mathrm{d}c}{\mathrm{d}\eta} + \sigma\frac{k_T}{T}\frac{\mathrm{d}T}{\mathrm{d}\eta}\right)\right] + \frac{f}{2}\frac{\mathrm{d}c}{\mathrm{d}\eta} = 0 \tag{8}$$

$$\frac{1}{\bar{c}_p}\frac{\mathrm{d}}{\mathrm{d}\eta}\left[\lambda\frac{\bar{c}_p}{Pr}\frac{\mathrm{d}T}{\mathrm{d}\eta}\right] + \frac{f}{2}\frac{\mathrm{d}T}{\mathrm{d}\eta} + \frac{\lambda Le}{Pr}\left[\frac{\mathrm{d}c}{\mathrm{d}\eta} + \sigma\frac{k_T}{T}\frac{\mathrm{d}T}{\mathrm{d}\eta}\right] \cdot \frac{c_{p\mathrm{He}} - c_{p\mathrm{N_2}}}{\bar{c}_p}\frac{\mathrm{d}T}{\mathrm{d}\eta} = 0. \tag{9}$$

The boundary conditions for the shear layer are

$$\eta = -\infty: \quad c = c_2 \quad T = T_2 \qquad f' = 0$$

$$\eta = 0: \qquad f = 0 \quad c' = -\sigma \frac{k_T}{T} T'$$

$$\eta = +\infty: \quad c = c_1 \quad T = T_1 \qquad f' = 1$$

The boundary conditions at $\eta = 0$ are derived from the facts that the mass flowing within the dividing streamtube ($\eta = 0$ for $r = r_0$) is constant and that in steady state the net transport of helium across that streamtube will vanish. In order to avoid lengthy numerical calculations, $Pr = Le = \lambda = 1{\cdot}0$ were used. This is to be compared with typical values (for $c_1 = 0{\cdot}18$) which are

$$Pr_1 = 0{\cdot}39 \qquad Pr_2 = 0{\cdot}42$$

$$Le_1 = 0{\cdot}67 \qquad Le_2 = 0{\cdot}68$$

$$\rho_1\mu_1/\rho_2\mu_2 = 1{\cdot}47.$$

The approximation will be discussed below.

As the separation effects are expected to be small, c_1 will not differ much from c_2 and $\bar{c}_p$ and σk_T can be assumed to be constant. The latter simplification was evaluated by a calculation of $c_2 - c_1$ based first upon σk_T and then upon $\sigma_0 k_T$. For $c \approx 0{\cdot}7$ the error introduced was less than 2 per cent. Furthermore, it is in the nature of the present problem that the diffusion transports are small (and cancel exactly on the streamline $\eta = 0$). Therefore the third term in equation (9) can be neglected and equations (7)–(9) reduce to

$$f''' + \frac{f}{2} f'' = 0 \tag{10}$$

$$c'' + \frac{f}{2} c' + \sigma_0 k_T \left(\frac{T'}{T}\right)' = 0 \tag{11}$$

$$T'' + \frac{f}{2} T' = 0. \tag{12}$$

The solution of equation (10) satisfying the above boundary conditions is available from Christian [5]. The solution of equation (12) is

$$T = T_2 + (T_1 - T_2) \,.\, f' \tag{13}$$

and only equation (11) need be integrated numerically. Results are shown in Fig. 3.

In order to get some feeling for the effect of the approximations implied by equations (10)–(12), the case for $Le = 1$, $Pr \to \infty$ was also considered. Physically this means that both the temperature and the concentration fields will vary in a layer (near $y = 0$) which is much thinner than the shear layer. Introducing a new indepdendent variable

$$\bar{\eta} = (\sqrt{Pr})\,(\sqrt{f_0'}) \cdot \eta$$

with $f_0' = 0{\cdot}587$, and using again $Le = \lambda = 1$, $\bar{c}_p$ and σk_T constant, equations (8) and (9) reduce to forms identical to (11) and (12) but with f replaced by $\bar{\eta}$ and the prime referring to differentiation with respect to the latter.

The boundary conditions after equation (9) now require $f' = 0{\cdot}587$ for all η in place of the previous f, f' conditions.

For $(T_1 - T_2)/T \ll 1$ the solutions to the energy and concentration relations are now

$$\left.\begin{aligned} T &= \frac{T_1 + T_2}{2} + \frac{T_1 - T_2}{2} \frac{1}{\sqrt{\pi}} \int_0^{\bar{\eta}} \exp\left(-\frac{\bar{\eta}^2}{4}\right) \mathrm{d}\bar{\eta} \\ c &= \frac{c_1 + c_2}{2} + \frac{1}{2\sqrt{\pi}} \sigma_0 k_T \frac{T_1 - T_2}{T} \int_0^{\bar{\eta}} \left(\frac{\bar{\eta}^2}{4} - 1\right) \cdot \exp\left(-\frac{\bar{\eta}^2}{4}\right) \mathrm{d}\bar{\eta} \end{aligned}\right\} \tag{15}$$

This result also is plotted in Fig. 3 and indicates that $(c_2 - c_1)$ is only 17 per cent larger than the corresponding case with $Pr = 1$, this despite the fact that a radically different flow field exists for the two cases. In one case ($Pr = 1$) the velocity varies from 0 to 1, while it remains constant over the region of importance for $Pr = \infty$ This insensitivity to changes in the flow field is due to the fact that heat and mass are transported in the same way by convection [u and v appear only in the convection terms on the left-hand sides of equations (4) and (6)] and that it is the ratio c'/T' which is important for mixture

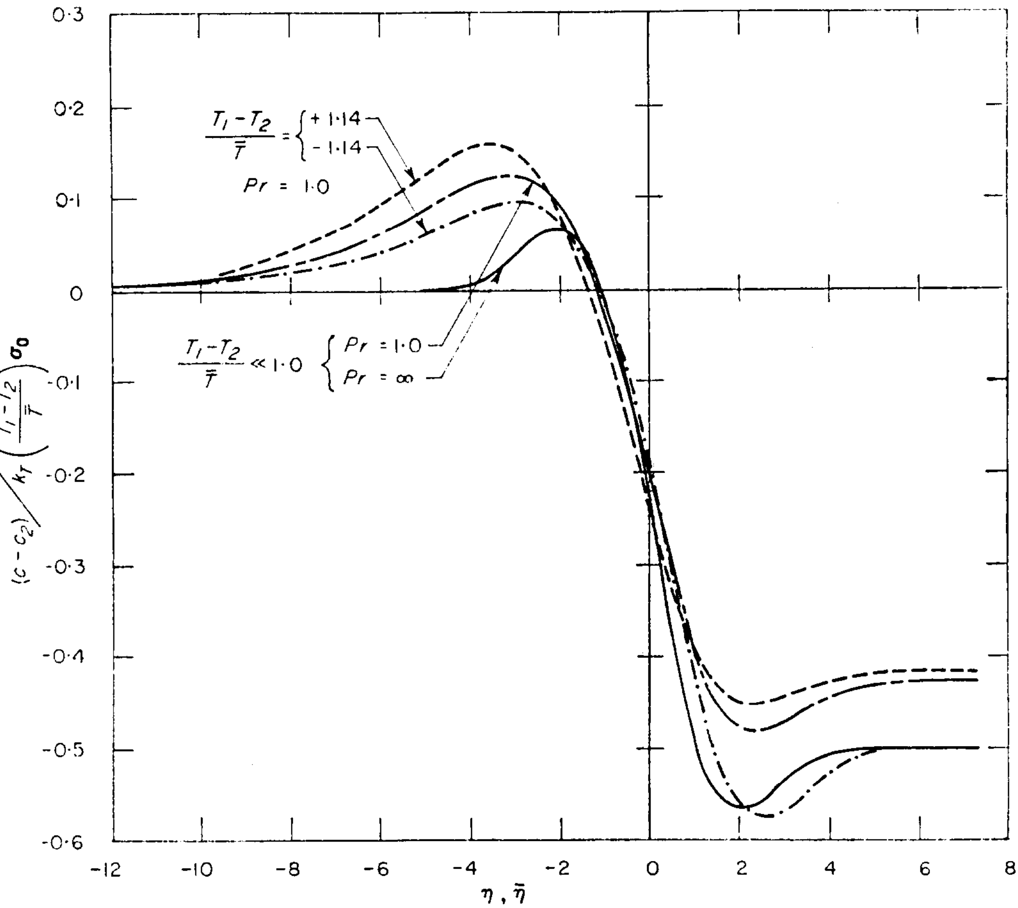

FIG. 3. Concentration distributions within the laminar shear layer, $Le = 1 \cdot 0$.

separation. The above analysis for a cooled laminar jet then predicts

$$c_2 - c_1 \cong -0 \cdot 5\, \sigma_0 k_T \frac{T_2 - T_1}{\bar{T}} \tag{16}$$

in place of equation (1). Although for the actual test Pr, Le and λ deviate appreciably from unity, equation (16) does furnish the correct order of magnitude for the coupling effect.

In the case of turbulent shear layer D^t and k_T^t are of different orders of magnitude, leading to different laws for the growth of the shear layer. However, the balance between thermal diffusion and concentration diffusion which determines $(c_2 - c_1)$ is the same as in the laminar case. It is therefore expected that equation (2) will also be corrected by a factor of approximately two, unless k_T^t or D^t vary in an extreme manner across the shear layer. This appears to be unlikely since the assumption of constant eddy viscosity leads to velocity profiles which are in good agreement with experiments (see, e.g. Schlichting [6]).

4. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. It consisted essentially of a cooled loop and a container at room temperature. The cooler contained liquid nitrogen such that the mixture passing through it emerged at 78°K. It then passed through a settling chamber and nozzle. Between the nozzle and a downstream intake the flow formed a free axisymmetric shear layer across which the individual components could be exchanged with those in the container (see Section 2). The container consisted of a Lucite cylinder, the ends of which were closed by means of two end plates. The plate at the upper end carried a 3·2 mm thick copper plate with an embedded heater sufficient to replace the heat lost by the container to the cooled loop. A second heater was located at the bottom of the container. When the jet was operated so as to be laminar, only the upper heater was used, as free convection induced by the lower heater disturbed the sensitive laminar jet.

The concentrations in the container and the jet were measured at the locations marked with c_1 and c_2 in Fig. 1. Gas samples were then passed through a constant temperature bath, concentration meter, diaphragm pump, flow meter and cold trap and finally back into either the container or the cooled loop. Both the concentration meter and the constant temperature bath were carefully insulated. Usually their temperature did not change more than 0·2 degC per h.

The concentration meter is described in detail by Brown [7]. It consists essentially of a tube, 15 in. long and 1 in. inner diameter. A microphone was located at each end of the tube, one being used to generate a sound field while the output of the other was a measure of the sound field amplitude. By varying either the tube length or the frequency, the system can be tuned to maximum output conditions. Such resonant conditions then imply the speed of sound on the basis of calibrations for length and the frequency. Together with the measured temperature of the sample, the concentration of helium can be calculated. Since the concentrations c_1 and c_2 were expected to differ by only a few per cent, some care was taken to measure the small difference with reasonable accuracy. To do so, the length of the resonance tube was kept constant and the frequency was varied. A counter was used to measure the frequency to an accuracy of 2 parts in 10 000. As it was difficult to determine the frequency of the maximum output (V_{max}) to a sufficient degree, the output voltage V was measured as a function of the frequency n. Near the maximum output $\sqrt{(V_{max} - V)}$ was quite accurately a linear function of n and this relation proved suitable to determine $n(V_{max})$. As the temperature of the concentration meter was essentially constant during the time samples were taken, the repeatability of the readings corresponded to $\pm$ 0·02 per cent of helium concentration.

The temperature in the cold loop was measured in the settling chamber; the temperature in the container was measured near the heated upper copper plate. In both cases iron-constantan thermocouples were used.

The jet velocity was calculated from the pressure drop across the nozzle evaluated with an oil manometer readable to 0·001 in. of oil. For the smallest speeds an extrapolation of Δp versus rev/min of the motor was employed, the rev/min being measured with a stroboscope.

The test procedure was as follows. The container was filled with a mixture of helium and nitrogen, each being passed through a cold trap. Then the blower was started, the cooler filled with nitrogen and the heater turned on. Within a few minutes the temperatures came to equilibrium. In the laminar case the concentration difference would then build up to a steady state within 10–15 min. Upon increasing the speed so as to achieve a turbulent jet, the concentration difference was found to decrease at a faster rate than was readable on the concentration meter, i.e. asymptotically within less than 1 min. Once equilibrium was attained, c_1 and c_2 were read alternately until the difference could be determined within the desired accuracy.

5. RESULTS AND DISCUSSION

The experimental setup was checked first by operating the jet with an uncooled mixture. The result is shown in the upper part of Fig. 4. Both c_1 and c_2 agreed with the mean value within $+$ 0·01 per cent of helium concentration. The jet was then run cooled but with nitrogen only and again c_1 and c_2 were equal within $+$0·01 per cent (lower part of Fig. 4). In the laminar case the jet was heated only from above. Therefore, the gradients of temperature and concentrations normal to the heated plate at the top were determined. By introducing smoke into the container it was observed that the streamlines passing near the probe and thermocouple form

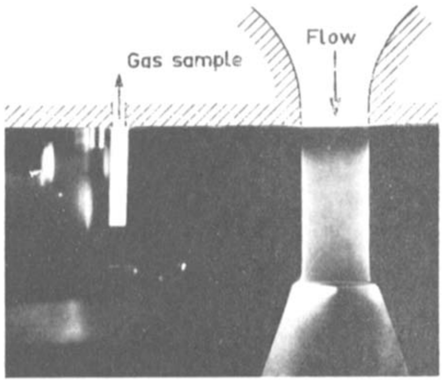

FIG. 6. Laminar jet, $T_1 = 79$ K, $T_2 = 328$ K, $\bar{c}_{He} = 0{\cdot}87$, $Re_L = 4 \times 10^3$.

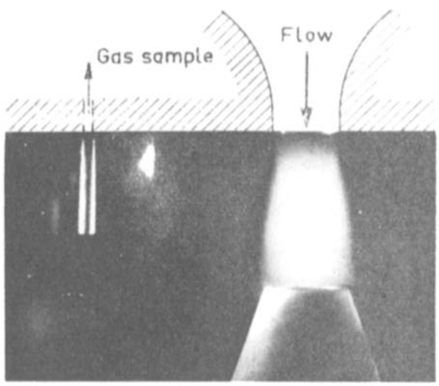

FIG. 7. Turbulent jet, uncooled, $\bar{c}_{He} = 0$, $Re_L = 10^5$.

the outer edge of the jet. The result of the concentration measurements are shown in Fig. 5 where it is seen that both the variation in c and T are small when compared with $(c_2 - c_2)$ and $(T_2 - T_1)$ respectively.

When the jet was cooled, water vapor was found to condense in the jet and showed up as fog. Such condensate usually appeared when the temperature dropped below 180 to 130°K, indicating a very low moisture content in the gas mixture. No correlation between the temperature at which the fog formed and the concentration difference was found. The fog proved to be quite convenient, making possible a continuous visual observation of the jet and indicating either the slightest misalignment between jet and intake or an unstable jet condition. Figure 6 shows a picture of the cooled laminar jet taken during one of the actual runs. Some indication of the steadiness of the flow is obtained by noting that the exposure time of the picture was 20 μs.

In the turbulent case it was necessary to introduce smoke for jet visualization, as the ice crystals were insufficiently dense due to their transport normal to the dividing streamtube by the turbulent diffusion process. Figure 7 shows

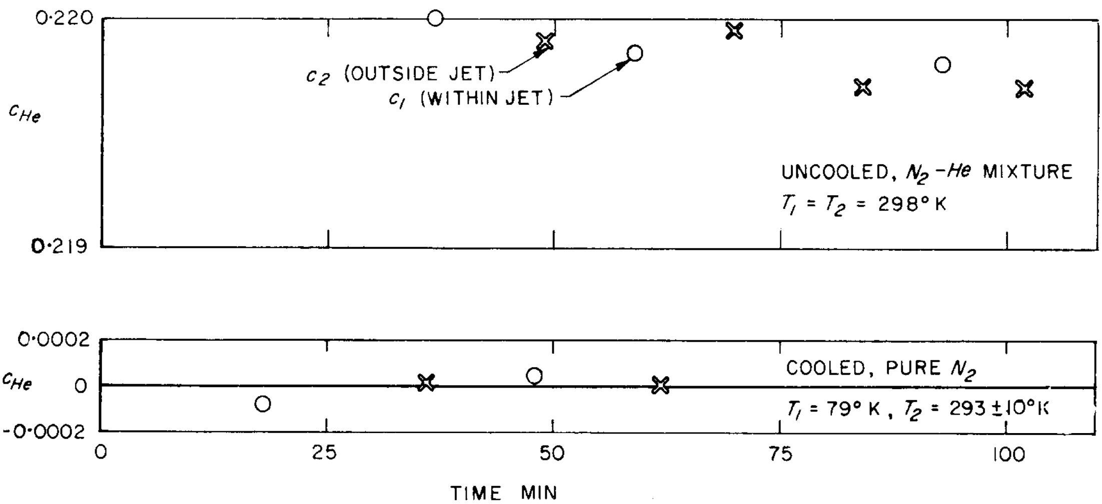

FIG. 4. Experimental concentration checks on operation without thermal diffusion effects. (a) laminar test, $Re_L = 1{\cdot}6 \times 10^3$, for $t < 75$ min; turbulent test, $Re_L = 4 \times 10^4$, for $t > 75$ min; (b) turbulent test, $Re_L = 10^5$.

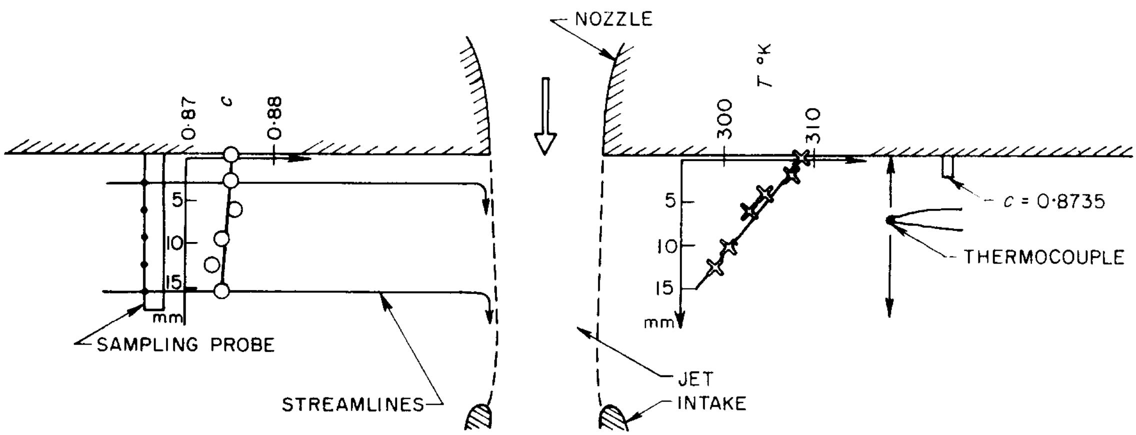

FIG. 5. Concentration and temperature distributions normal to the upper wall, laminar jet.

a picture of an uncooled turbulent jet. The transition from a laminar to a turbulent shear layer is clearly seen. In the laminar part of the shear layer the vorticity diffuses much faster than the smoke particles (Brownian motion). At the transition point the laminar shear layer is therefore much wider than indicated by the smoke layer. After transition, the smoke is moved by turbulent diffusion and in a short length fills the entire width of the shear layer. This explains the apparent kink in the visible outer edge of the shear layer in Fig. 7. Due to experimental difficulties and lack of time a good picture of the cooled turbulent jet could not be taken, but it was observed that the transition point occurred at about 0·2 nozzle diameters from the exit at the highest Reynolds number.

In Fig. 8 the measured concentration differences are compared with the theoretical result for a thin laminar shear layer. Taking into account that Le, Pr and $(\rho\mu/\rho_1\mu_1)$ differ from unity, that the shear layer in fact is not thin compared with the radius, and that the effects of free convection were neglected, the agreement between theory and experiment is good. A concentration profile was measured in the radial direction. The result is shown in Fig. 9 and indicates that the shear layer is indeed no longer thin compared with the radius. This direct measurement of $(c_2 - c_1)$ with the probe also is indicated as a triangle in Fig. 8 and is in good agreement with the previous results.

In the turbulent case, a very small concentration difference was measurable (Fig. 8). The results were identical regardless of the initial level (smaller, same, or larger) of helium in the jet before steady state was achieved. As the difference $(c_2 - c_1)$ has opposite signs in the laminar and turbulent cases, the difference cannot be due to the short length of laminar

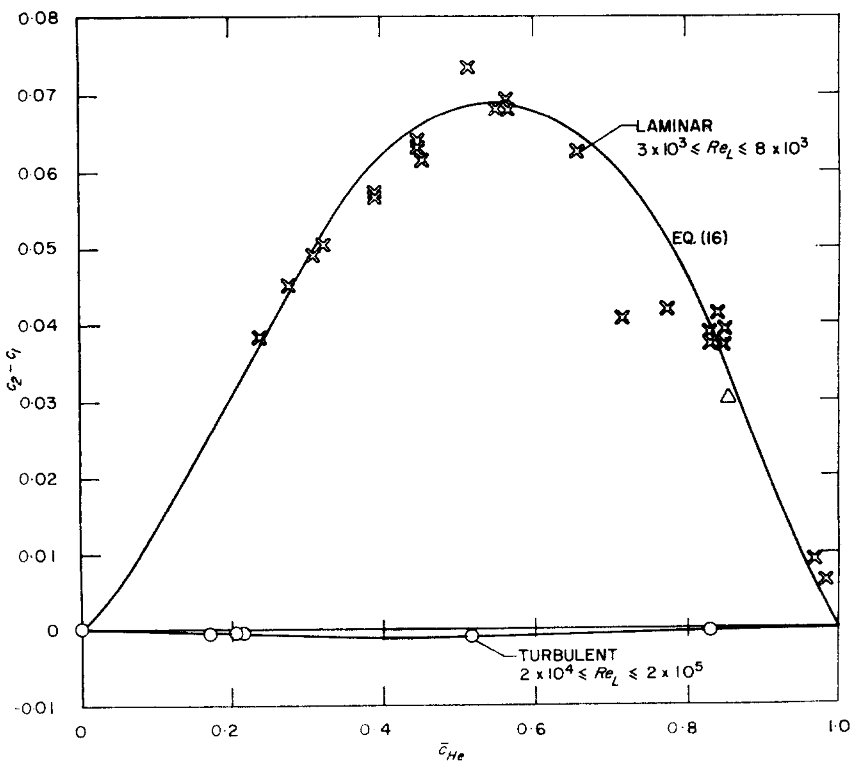

FIG. 8. Mixture separation for laminar and turbulent shear layers, $T_1 = 78°K$, $T_2 = 310°K$.

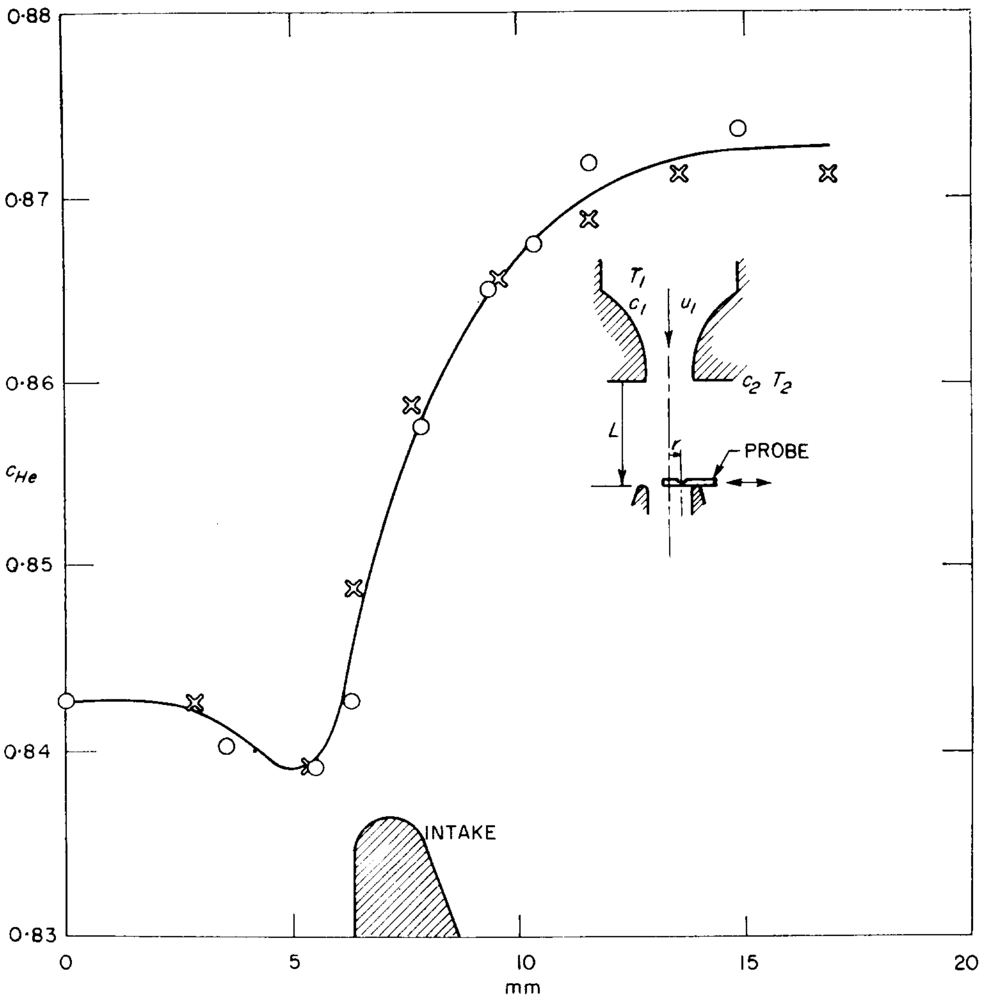

FIG. 9. Concentration distribution in laminar jet shear layer, $Re_L = 4 \times 10^3$.

shear layer which was always present near the nozzle exit. As $(c_2 - c_1)$ was very small, any influence of Reynolds number was within experimental scatter if present at all.

6. CONCLUSIONS

For a cooled recirculating laminar jet the concentration difference between the jet and the surroundings amounted to about 7 per cent. This value was in reasonable agreement with a simplified analysis.

Similar separation effects can also be expected for other steady flows with closed streamlines such as laminar base flows and flows in cavities.

For a turbulent jet, the concentration difference was at least two orders of magnitude smaller than in the laminar case and of opposite sign.

ACKNOWLEDGEMENTS

This research was sponsored by the U.S. Air Force Office of Scientific Research under Grant No. 62-407. Mr. Thomann was on leave from the Aeronautical Research Institute of Sweden during the course of this study.

REFERENCES

1. J. R. Baron, Thermodynamic coupling in boundary layers, *J. Amer. Rocket Soc.* **32**, 1053–1959 (1962).
2. O. E. Tewfik, E. R. G. Eckert and C. J. Shirtliffe, Thermal diffusion effects on energy transfer in a turbulent boundary layer with helium injection, p. 42. *Proceedings 1962 Heat Transfer and Fluid Mechanics Institute*. Stanford University Press (1962).
3. F. E. C. Culick, The compressible turbulent boundary layer with surface mass transfer. Mass. Inst. of Tech., Naval Supersonic Laboratory, Rpt. 454, *AFOSR TN* 60-1094 (1960).
4. J. R. Baron, The binary-mixture boundary layer associated with mass transfer cooling at high speeds. Mass. Inst. of Tech., Naval Supersonic Laboratory, Rpt. 160 (1956).
5. W. J. Christian, Improved numerical solution of the Blasius problem with three-point boundary conditions, *J. Aero-Space Sci.* **28**, 911–912 (1961).
6. H. Schlichting, *Grenzschicht-Theorie*, p. 447. Karlsruhe (1951).
7. J. R. Brown, Design calibration and analysis of a helium concentration meter. Mass. Inst. of Tech., Naval Supersonic Laboratory, Rpt. **416**, *AFOSR TN* 60–39 (1959).
8. J. R. Baron, Thermal diffusion effects in mass transfer, *Int. J. Heat Mass Transfer* **6**, 1025–1033 (1963).

Zusammenfassung—Um die Grössenordnung der Thermodiffusion durch eine laminare und turbulente Schicht einer Scherströmung zu bestimmen, wurde eine experimentelle Untersuchung durchgeführt. Ein kurzes Stück eines gekühlten freien Strahles wurde durch ein ruhendes Gas geschickt und anschliessend in ein System mit kontinuierlichem Umlauf geleitet. Dabei wurden verschiedene Gemische von Helium und Stickstoff untersucht. Bei Temperaturen von 78°K im Strahl und 310 °K in der ihn umgebenden Kammer war die stationäre Konzentration des Heliums im laminaren Strahl um 7% geringer als in der Umgebung. Dieser Wert steht in tragbarer Ubereinstimmung mit einer vereinfachten Analyse.

Bei einer turbulenten "Scherschicht" zwischen dem Strahl und der Umgebung steigt die Heliumkonzentration im Strahl und liegt bis zu 0,1% über dem Kammerniveau. Das Thermodiffusionsverhältnis (d.h. das Verhältnis des Thermodiffusionskoeffizienten zum Massendiffusionskoeffizienten auf Grund der Konzentrationsunterschiede) in der turbulenten "Scherschicht" war somit um wenigstens zwei Grössenordnungen kleiner als im laminaren Fall und hatte entgegengesetztes Vorzeichen. Man darf vermuten, dass ähnliche Trenneffekte für andere stationäre Strömungen mit geschlossenen Stromlinien, wie Bodenströmungen und Strömungen hinter Hohlräumen, zu erwarten sind.

Аннотация—Проводилось экспериментальное исследование по определению порядка величины термодиффузии в ламинарном и турбулентном вязком потоке. Короткая холодная свободная струя пропускалась через стационарный газ и затем вновь возвращалась в непрерывно циркулирующую систему. Исследовались различные смеси гелия и азота. При температуре 78°К в струе и 310°К в камере равновесная концентрация гелия в ламинарной струе была на 7 процентов меньше, чем в окружающей среде. Экспериментальные данные хорошо согласуются с проведенным упрощенным анализом.

Если между струей и окружающей средой находится турбулентный пограничный слой, концентрация гелия внутри струи увеличивается на 0,1% по сравнению с концентрацией в камере. Термодиффузионное отношение в турбулентном пограничном слое по крайней мере на два порядка меньше, чем в ламинарном, и имеет противоположный знак. Высказывается предположение, что подобное влияние срыва потока следует ожидать в других случаях установившихся течений с замкнутыми линиями тока, таких как донные и кавитационные течения.